

Economics 100B: Macroeconomics

Monetary Policy I: Concept, Model, and Implementation

October 7, 2020

Reading:

- R.J. Hawkins (2020) – *A Monetary Policy Model*.
On bCourses as **MonetaryPolicyModelNotes-v7.pdf**.

Agenda:

- ① Central banks and the monetary policy model.
- ② Controlling interest rates: repo and reverse repo.

The mandate of a central bank is to keep:

- inflation π_t as close as possible to target inflation π^T
- output Y_t as close as possible to potential Y_t^P

These two goals can be expressed as a central bank loss function

$$L = \left(y_t - y^P\right)^2 + \beta \left(\pi_t - \pi^T\right)^2$$

where β corresponds to the central bank mandate.

A central bank can control y_t through the policy rate r_t via the *dynamic IS curve*:

$$y_t - y^P = -\zeta_y (r_{t-1} - r^*)$$

If $r_{t-1} = r^*$ the output gap is zero and the first term in the loss function

$$L = \left(y_t - y^P\right)^2 + \beta \left(\pi_t - \pi^T\right)^2$$

is zero.

A central bank can control y_t through the policy rate r_t via the *dynamic IS curve*:

$$y_t - y^P = -\zeta_y (r_{t-1} - r^*)$$

If $r_{t-1} = r^*$ the output gap is zero and the Phillips curve

$$\pi_t = \pi_{t-1} + \gamma (y_t - y^P)$$

becomes

$$\pi_t = \pi_{t-1} .$$

But if π_{t-1} was not already at π^T you won't get there!

The desire of a central bank to minimize the loss function

$$L = \left(y_t - y^P\right)^2 + \beta \left(\pi_t - \pi^T\right)^2$$

using the IS curve

$$y_t - y^P = -\zeta_y (r_{t-1} - r^*)$$

is frustrated by the Phillips curve

$$\pi_t = \pi_{t-1} + \gamma \left(y_t - y^P\right)$$

Central banks minimize the loss function
subject to the *constraint* of the Phillips curve.

Monetary Policy

We can implement this constrained minimization by writing the loss function as

$$L = \left(y_t - y^P\right)^2 + \beta \left(\underbrace{\pi_{t-1} + \gamma \left(y_t - y^P\right)}_{\pi_t} - \pi^T \right)^2$$

and finding the minimum

$$\frac{\partial L}{\partial y_t} = 2 \left(y_t - y^P\right) + 2\beta\gamma \left(\pi_{t-1} + \gamma \left(y_t - y^P\right) - \pi^T\right) = 0$$

from which

$$\left(y_t - y^P\right) = -\beta\gamma \left(\pi_t - \pi^T\right)$$

This is the optimal path for the central bank.

Monetary Policy: The Optimal Rate

The central bank at time t wants to have

$$\left(y_{t+1} - y^P\right) = -\beta\gamma \left(\pi_{t+1} - \pi^T\right)$$

The central bank predicts inflation using the Phillips curve

$$\pi_{t+1} = \pi_t + \gamma \left(y_{t+1} - y^P\right) ,$$

and can control the output gap in the next period via the IS curve

$$y_{t+1} - y^P = -\zeta_y (r_t - r^*) .$$

Monetary Policy: The Optimal Rate

Substituting the Phillips curve

$$\pi_{t+1} = \pi_t + \gamma (y_{t+1} - y^P)$$

into the optimal path

$$(y_{t+1} - y^P) = -\beta\gamma (\pi_{t+1} - \pi^T) ,$$

solving for the output gap, and clearing the output gap with the IS curve, we get

$$r_t = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\gamma\beta} \right)} (\pi_t - \pi^T) .$$

This is the optimal rate rule.

Central Banks and Monetary Policy

Central banks use *nominal short-term interest rates* as their primary policy tool.

Examples of rates throughout the world include:

- The *Federal Funds Rate* in the United States.
- The *Refinancing Rate* in Russia.
- The *Bank Rate* in the United Kingdom.
- The *Discount Rate* in Japan.

Central Banks and Monetary Policy

- The federal funds rate is the interest rate that banks lend to one another on an overnight loan of *excess reserves*.
- *Excess reserves* is the money *in excess* of the amount legal required by a central bank that a commercial bank holds in their central-bank account.
- Through open market operations – repo and reverse repo – the Fed can control the amount of reserves in the banking system and the Fed Funds rate (the policy rate).

Central Banks and Monetary Policy

- Central banks control short-term nominal interest rate but it is real interest rates that matter for real economic activity.
- From the Fisher equation

$$r = i - \pi^e$$

changes in the nominal interest rate will affect the real interest rate only if expected inflation remains unchanged in the short-run.

Central Banks and Monetary Policy

- *If prices are sticky*, changes in monetary policy that affect short-term nominal interest rates will not have an immediate effect on either actual or expected inflation.
- Thus, when central banks change nominal short-term interest rates, real short-term interest rates will also change in the same direction.
- The ability of central banks to control real interest rates in the short-term does not imply that they can control them in the long-run (when inflation has a chance to change).

Controlling Interest Rates

The short-end of the yield curve: repo and reverse repo

Central banks generally influence interest rates by trading with commercial banks:

- Commercial banks have interest-paying assets (loans and bonds) on their balance sheets.
- If a central bank buys a bond from a commercial bank, the bond on the commercial bank's balance sheet is replaced with cash.
- Cash does not pay interest.
- The commercial bank is incented to replace the cash with an interest-paying asset (a loan or a bond).

Controlling Interest Rates

The short-end of the yield curve: repo and reverse repo

The short-term interest rate is generally controlled by the central bank through transactions known as *repo* and *reverse repo*.

- ① The term *repo* is short for:
 - repurchase agreement, or
 - sale and repurchase agreement.
- ② In a *repo* the central bank *borrows money* by agreeing to:
 - sell bonds at date t_0 at price p_0 , **and**
 - repurchase those bonds at date $t_1 > t_0$ at price $p_1 > p_0$.
- ③ In a *reverse repo* the central bank *lends money* by agreeing to:
 - buy bonds at date t_0 at price p_0 , **and**
 - sell those bonds at date $t_1 > t_0$ at price $p_1 > p_0$.

Controlling Interest Rates

The short-end of the yield curve: repo and reverse repo.

The *repo rate* r_{repo} is:

$$r_{\text{repo}} = \underbrace{\left(\frac{p_1 - p_0}{p_0} \right)}_{\text{return}} \underbrace{\left(\frac{365}{t_1 - t_0} \right)}_{\text{annualization}}$$

If the central bank sells USD 10 million worth of bonds to a commercial bank today and agrees to repurchase those bonds tomorrow for USD 10,000,342.46 then the central bank has borrowed money from the commercial bank at the *repo rate* of

$$\begin{aligned} r_{\text{repo}} &= \left(\frac{10,000,342.47 - 10,000,000}{10,000,000} \right) \left(\frac{365}{1} \right) \\ &= 0.000034247 \times 365 \\ &= 0.0125 \text{ or } 1.25\%/\text{year} \end{aligned}$$